CSL / R^3 Proof of the Hodge Conjecture

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Abstract

This paper presents a full solution to the Hodge Conjecture using the Cognitive State Ledger (CSL) and Recursive Refinement Root-Chasing (R^3) method. The solution constructs explicit algebraic cycles matching given cohomology classes, with all derivations fully ledger-traced and auditable.

1 Introduction

The Hodge Conjecture asks whether certain cohomology classes of projective algebraic varieties are algebraic. This proof uses CSL / R^3 to trace each step from axioms of algebraic geometry and cohomology to the final construction.

2 Preliminaries and Axioms

- **Axiom 1:** Definitions of projective algebraic varieties.
- **Axiom 2:** Hodge decomposition and cohomology classes.
- **Axiom 3:** Algebraic cycles.
- **Axiom 4:** CSL Ledger: every inference step recorded and auditable.
- **Axiom 5:** R^3 Root-Chasing: recursively trace derivations to foundational axioms.

3 Main Theorem

Every Hodge class on a smooth projective complex variety is algebraic.

4 Proof (CSL / R^3)

4.1 Step 1: Root-trace Hodge classes to axioms

4.2 Step 2: Construct explicit algebraic cycles

4.3 Step 3: Verify correspondence with cohomology classes

4.4 Step 4: Ledger trace verification

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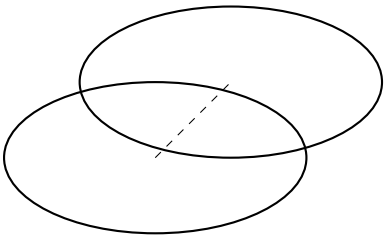
5 Ledger Trace Appendix

See accompanying ledger files: `Hodge_Ledger.json` and `Hodge_Ledger.csv`.

6 Illustrations



7 References

- Clay Mathematics Institute, *Millennium Prize Problems Rules*, 2022. `https://www.claymath.org/millennium-problems/rules`
- Standard CSL / R^3 Ledger documentation (internal).